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Software technology for automated feedback generation

Bastiaan Heeren, ICS Colloquium December 17, 2020

Short bio

- Associate professor at the **Open University of the Netherlands**
- Head of OU's Computer Science Department

- At **Utrecht University**:
 - PhD in Software Technology (2000-2005)
 - Lecturer (2005-2007)
 - Guest researcher with the Software Technology for Learning and Teaching research group



Bastiaan Heeren

Intelligent Tutoring Systems

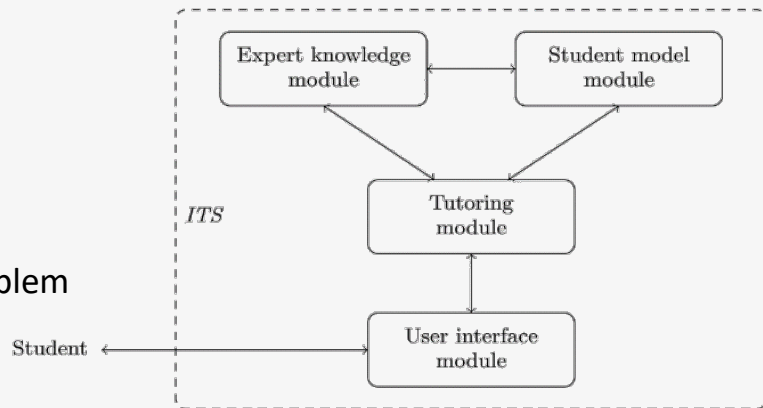
Intelligent Tutoring Systems (ITS): computer systems that provide immediate and customized feedback to learners

Structure:

- Classical architecture with **four components**

Behaviour:

- **Outer loop**: solving one task after another
- **Inner loop**: the steps for solving one complex, multi-step problem



Example: axiomatic proofs



Ideas - LogAx

NL

EN

Help

Log out

Axiomatic

New exercise (N)



1	$p \vdash p$	Assumption	X
2	$p \rightarrow q \vdash p \rightarrow q$	Assumption	X
998	$p, p \rightarrow q, q \rightarrow r \vdash r$		X
999	$p \rightarrow q, q \rightarrow r \vdash p \rightarrow r$	Deduction 998	X
1000	$q \rightarrow r \vdash (p \rightarrow q) \rightarrow (p \rightarrow r)$	Deduction 999	

multiple solutions are accepted

step-wise construction

Rule: Modus Ponens

$(\Sigma \vdash_S \varphi), (\Delta \vdash_S \varphi \rightarrow \psi) \Rightarrow \Sigma \cup \Delta \vdash_S \psi$

$\Sigma \vdash_S \varphi$ stepnr

$\Delta \vdash_S \varphi \rightarrow \psi$ stepnr

$\Sigma \cup \Delta \vdash_S \psi$ stepnr

Hint Next step Apply

Show complete derivation Complete my derivation

feedback and hints



Research motivation

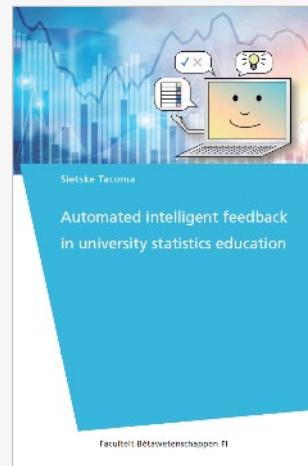
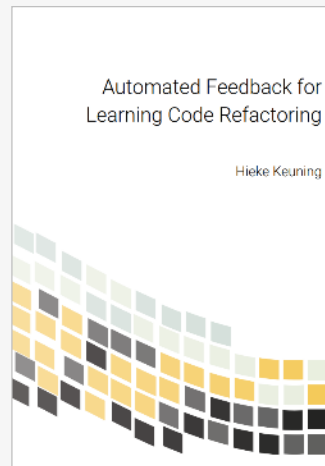
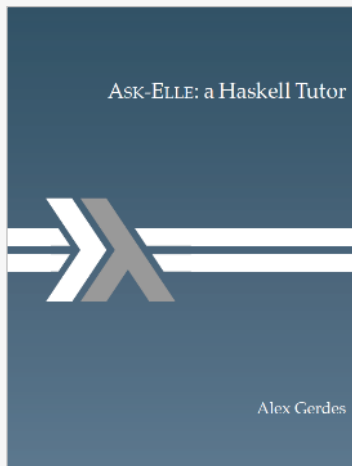
1. Simplify construction of ITSs (which are complex software systems)
2. Represent expert domain knowledge explicitly (for better feedback)
3. Apply approach to a wide range of problem domains

Approach: use software technology for automated feedback generation

Techniques in this presentation (outline):

- Rewrite strategies for automated feedback (basics)
- Light-weight rewrite rules
- Generic traversals

Problem domains



Four recent PhD theses, for different problem domains, all based on the same approach



Ideas framework

Generic framework for constructing domain reasoners

- Developed in Haskell
- Size: 12,397 LOC
- Open source
- Independent of problem domain
- <http://ideas.cs.uu.nl/tutorial/>



Interactive Domain-specific Exercise Assistants

» Hackage :: [Package] Search · Browse · What's new · Upload · User accounts

ideas: Feedback services for intelligent tutoring systems

[apache, education, library] [Propose Tags]

Ideas (Interactive Domain-specific Exercise Assistants) is a joint research project between the Open University of the Netherlands and Utrecht University. The project's goal is to use software and compiler technology to build state-of-the-art components for intelligent tutoring systems (ITS) and learning environments. The `ideas` software package provides a generic framework for constructing the expert knowledge module (also known as a domain reasoner) for an ITS or learning environment. Domain knowledge is offered as a set of feedback services that are used by external tools such as the digital mathematical environment (DME), MathDox, and the Math-Bridge system. We have developed several domain reasoners based on this framework, including reasoners for mathematics, linear algebra, logic, learning Haskell (the Ask-Elle programming tutor) and evaluating Haskell expressions, and for practicing communication skills (the serious game Communicate!).

Modules

[Index]
Ideas

Common

- Ideas.Common.Classes
- Ideas.Common.Constraint
- Ideas.Common.Context
- Ideas.Common.Derivation
- Ideas.Common.DerivationTree

Versions [faq]
0.5.8, 0.6, 0.7, 1.0, 1.1, 1.2, 1.3, 1.3.1, 1.4, 1.5, 1.6, 1.7, 1.8

Change log
[CHANGELOG.txt](#)

Dependencies

base (>=4.8 && <5), blaze-builder (>=0.4), bytestring, case-insensitive, containers, Diff, directory, filepath, HDBC, HDBC-sqlite3, http-types, mtl, network, parsec, QuickCheck (>=2.8 && <2.12), random, semigroups (==0.18.*), streaming-commons (<0.2), time, uniplate, wai, wai-pprint [details]

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Category
Education

Home page
<http://ideas.cs.uu.nl/>

Source repo
head: git clone <https://github.com/ideas-edu/ideas.git>



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Rewrite strategies



Rewrite strategies for automated feedback

- Domain-specific language for specifying **problem-solving procedures**:
 - describe sequences of rule applications that solve a particular task
 - are formalized by a trace-based semantics (CSP)
 - allow new composition operators (interleaving, topological sorts)
- Problem-solving procedures are used for **feedback generation**:
 - recognizing the solution strategy
 - detecting detours
 - suggesting subgoals
 - providing next-step hints
 - providing worked-out examples



Example

Goal: rewrite proposition into
negation normal form (NNF)

$$\underline{\neg((p \vee q) \wedge \neg(p \wedge r))}$$

\Leftrightarrow De Morgan

$$\underline{\neg(p \vee q)} \vee \underline{\neg\neg(p \wedge r)}$$

\Leftrightarrow De Morgan

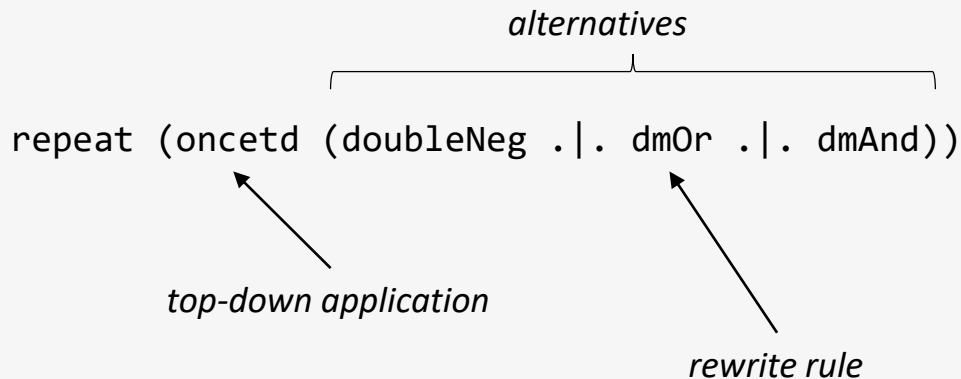
$$(\neg p \wedge \neg q) \vee \underline{\neg\neg(p \wedge r)}$$

\Leftrightarrow Double Neg

$$(\neg p \wedge \neg q) \vee (p \wedge r)$$



Rewrite strategy for NNF:





Strategy combinators

<code>p .* q</code>	sequence: first p, then q
<code>succeed</code>	always succeeds
<code>p q</code>	choice: p or q
<code>p ./ q</code>	preference: p is preferred over q
<code>p > q</code>	left-biased choice: p or else q
<code>fix</code>	fixed-point combinator

Derived combinators:

```
try s = s |> succeed
```

```
repeat s = try (s .* repeat s)
```

Finite representation with explicit recursion:

```
repeat s = fix $ \x ->
  try (s .* x)
```

Advantages:

- Extract rules from strategy
- Customize strategy
- Document/visualise strategy



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Light-weight rewrite rules



Proposition logic

```
data Logic = Logic :&&: Logic  -- conjunction
          | Logic :||: Logic  -- disjunction
          | Not Logic          -- negation
          | Var String         -- variable
```

Representation can be more complex, with nested and parameterised datatypes, e.g.:

$$3x + 9 = 0 \quad \forall x = 1$$

1	$p \vdash p$	Assumption
2	$p \rightarrow q \vdash p \rightarrow q$	Assumption
998	$p, p \rightarrow q, q \rightarrow r \vdash r$	
999	$p \rightarrow q, q \rightarrow r \vdash p \rightarrow r$	Deduction 998
1000	$q \rightarrow r \vdash (p \rightarrow q) \rightarrow (p \rightarrow r)$	Deduction 999



Rewrite rules

```
doubleNeg = rewriteRule "doubleNeg" $  
  \phi -> Not (Not phi) :~> phi
```

```
dmAnd = rewriteRule "dmAnd" $  
  \phi psi -> Not (phi :&&: psi) :~> Not phi :||: Not psi
```

*meta-variables are
introduced by lambdas*

left-hand side

right-hand side

$\neg\neg\phi \Leftrightarrow \phi$
$\neg(\phi \wedge \psi) \Leftrightarrow \neg\phi \vee \neg\psi$
$\neg(\phi \vee \psi) \Leftrightarrow \neg\phi \wedge \neg\psi$

How to use such rewrite rules?



Embedding-projection pair

Approach: conversion from/to a generic Term datatype with support for meta-variables

```
toTerm    :: Logic -> Term
fromTerm  :: Term  -> Maybe Logic
```

- From/to should be inverse functions (intuitively)
- Conversion allows generic functions, such as unification and zippers
- Pair can be derived automatically from the datatype definition

Note: more powerful generic programming libraries exist that can guarantee more type safety, with less overhead

Compiling rewrite rules

$\backslash\text{phi} \rightarrow \text{Not} (\text{Not } \text{phi}) : \sim \rightarrow \text{phi}$

*values provided by user
for problem domain*

Step 1: use two different values (e.g. Var “p” and Var “q”):

$\text{Not} (\text{Not} (\text{Var } \text{“p”})) : \sim \rightarrow \text{Var } \text{“p”}$

$\text{Not} (\text{Not} (\text{Var } \text{“q”})) : \sim \rightarrow \text{Var } \text{“q”}$

*TCon, TVar, and TMeta
are constructors of Term*

Step 2: convert to Term datatype:

$\text{TCon } \text{“Not”} [\text{TCon } \text{“Not”} [\text{TVar } \text{“p”}]] : \sim \rightarrow \text{TVar } \text{“p”}$

$\text{TCon } \text{“Not”} [\text{TCon } \text{“Not”} [\text{TVar } \text{“q”}]] : \sim \rightarrow \text{TVar } \text{“q”}$

Step 3: find meta-variables by comparing left-hand sides and right-hand sides

$\text{TCon } \text{“Not”} [\text{TCon } \text{“Not”} [\text{TMeta } \emptyset]] : \sim \rightarrow \text{TMeta } \emptyset$



Applying rewrite rules

Not (Not (Var “p” :&&: Var “r”))

Rewrite rule:

```
TCon “Not” [TCon “Not” [TMeta 0]]  
:~> TMeta 0
```

Step 1: convert to Term datatype:

```
TCon “Not” [TCon “Not” [TCon “And” [TVar “p”, TVar “q”]]]
```

Step 2: match with rule’s left-hand side:

```
0 = TCon “And” [TVar “p”, TVar “q”]
```

Step 3: substitute in rule’s right-hand side:

```
TCon “And” [TVar “p”, TVar “q”]
```

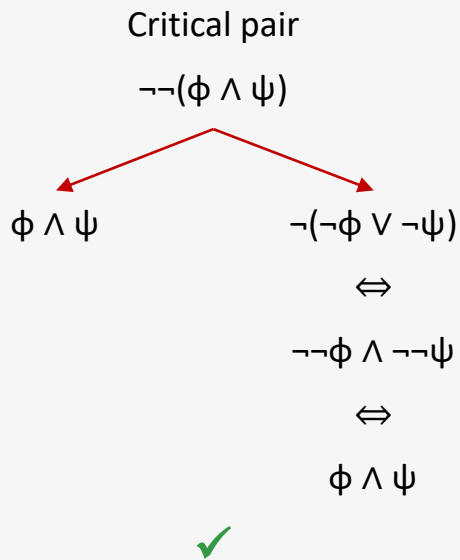
Step 4: convert back to Logic:

```
Var “p” :&&: Var “r”
```



Knuth-Bendix completion

Use case for explicit representation: search for missing rewrite rules (and reach confluence)



$$\neg\neg\phi \Leftrightarrow \phi$$

$$\neg(\phi \wedge \psi) \Leftrightarrow \neg\phi \vee \neg\psi$$

Missing rule:

$$\neg(\phi \vee \psi) \Leftrightarrow \neg\phi \wedge \neg\psi$$



Light-weight rewrite rules

Advantages of explicit representation:

- Knuth-Bendix completion (analysis)
- AC-rewriting
- Rule inversion
- Automated testing
- Documentation (pretty-printing)

Summary for rewrite rules:

- ✓ Simplify construction (light-weight embedding)
- ✓ Explicit representation (for better feedback)
- ✓ Many problem domains



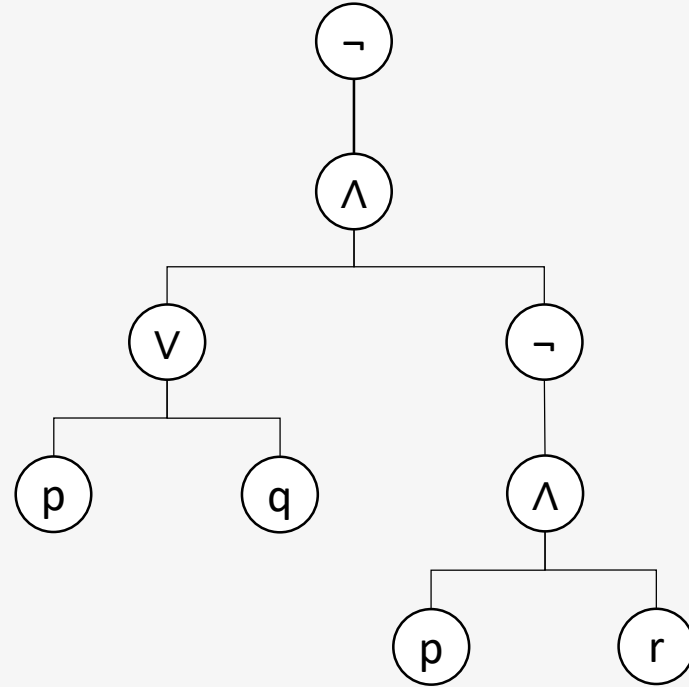
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Generic traversals



Tree representation

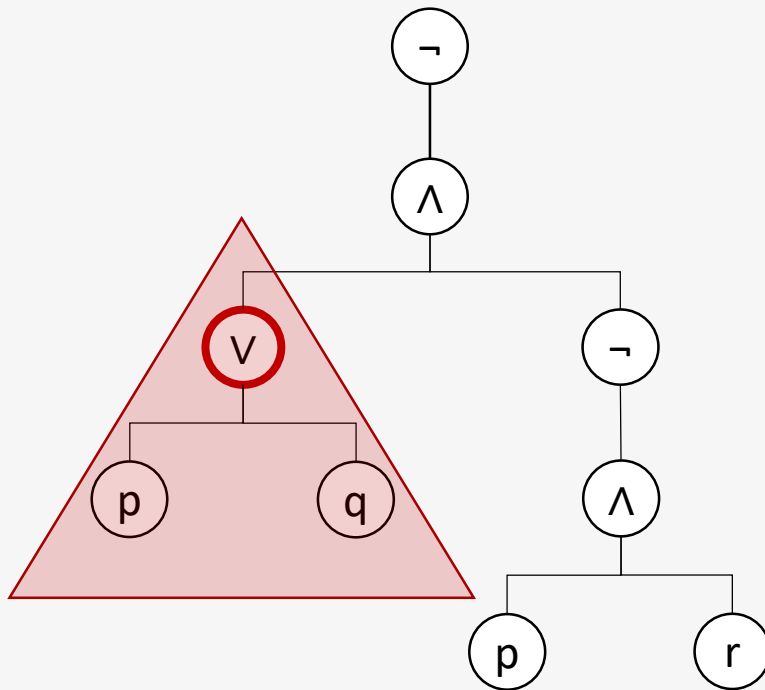
$\neg((p \vee q) \wedge \neg(p \wedge r))$



Point of focus

$$\neg((\underline{p \vee q}) \wedge \neg(p \wedge r))$$

- Implemented as a so-called zipper over the generic Term datatype
- Stored in a Context

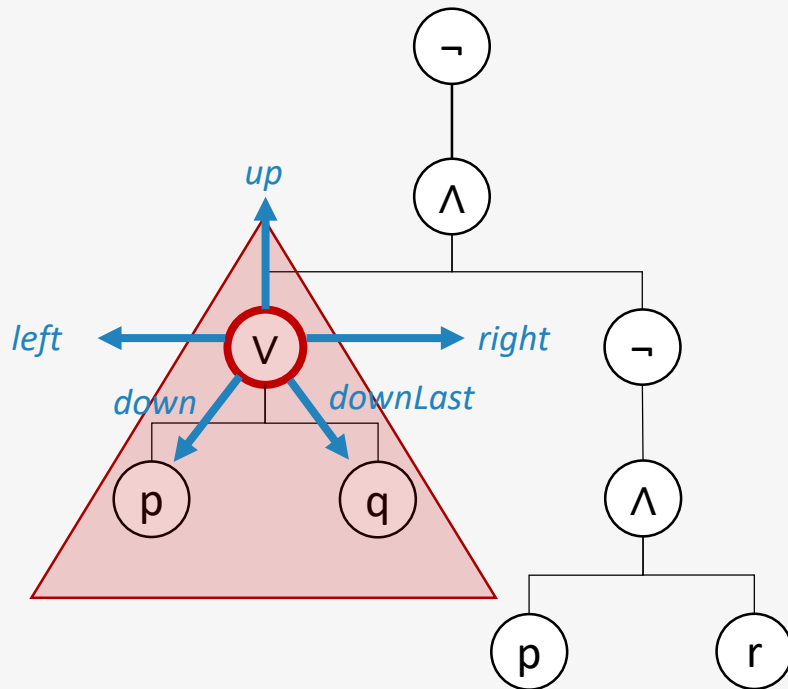


Navigation

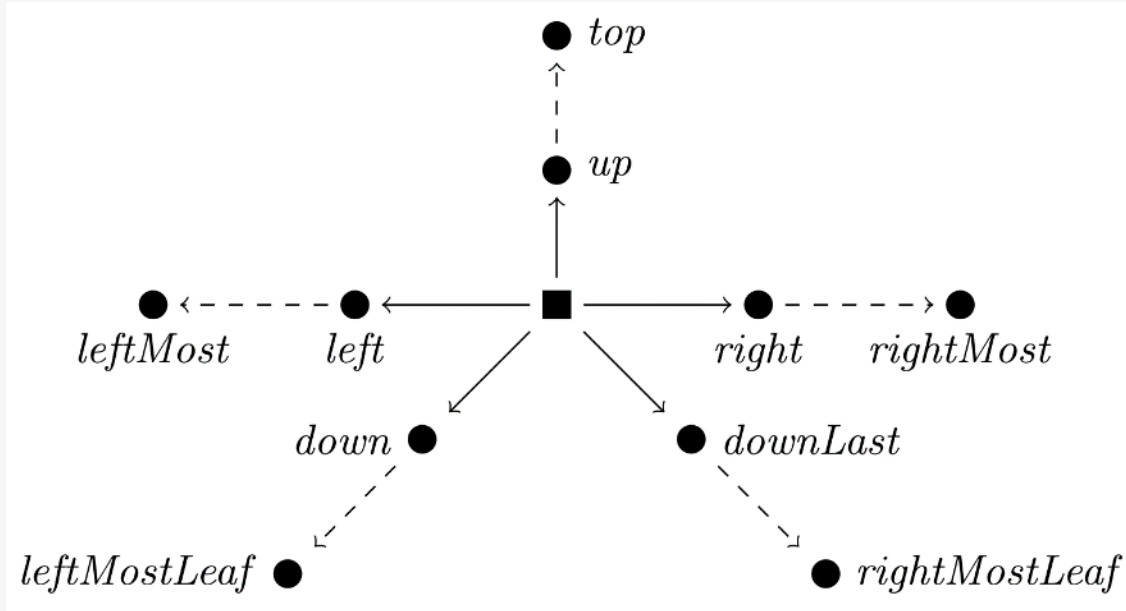
Five navigational actions:

- up
- left
- right
- down
- downLast

- Actions may fail
- Many useful laws, e.g.:
 - left \circ right \approx id
 - up \circ down \approx id



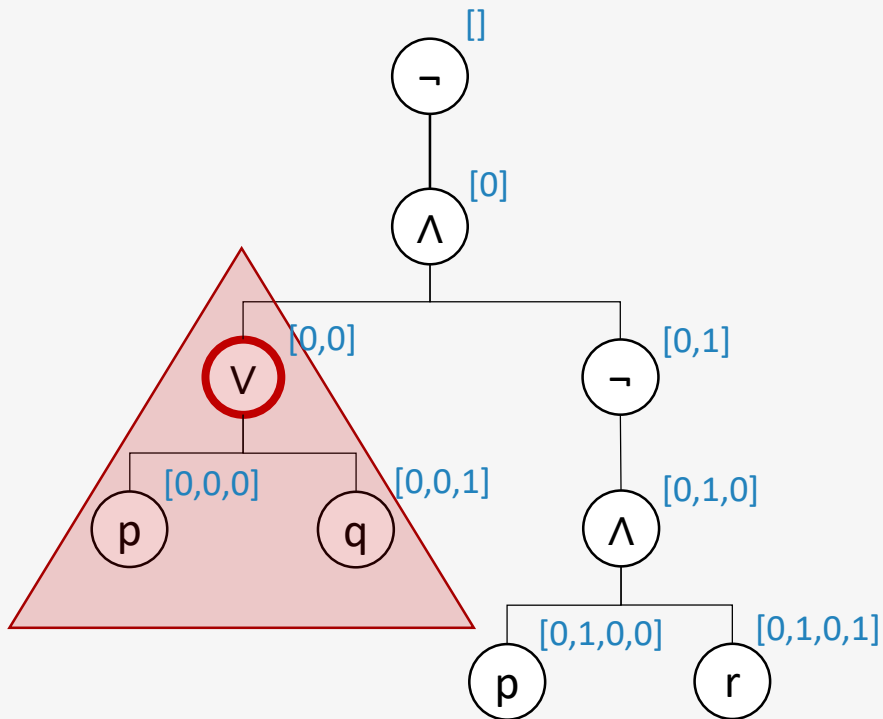
Navigation (extended)



From: [Traversals with Class](#). In Jurriaan Hage and Atze Dijkstra, editors, *Een Lawine van Ontwortelde Bomen: Liber Amicorum voor Doaitse Swierstra*, pages 62-75. 2013.

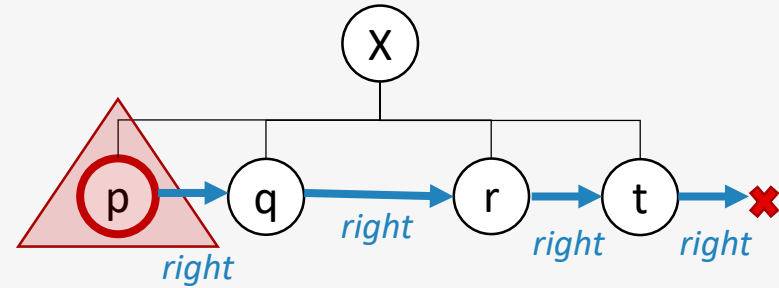
Position

- Zippers keep a position for the point of focus
- Position information is useful for generating feedback





Horizontal visits



```
visitOne s = fix $ \x -> s .|. (right .*. x)
```

```
visitFirst s = fix $ \x -> s |> (right .*. x)
```

```
visitAll s = fix $ \x -> s .*. (not right |> (right .*. x))
```

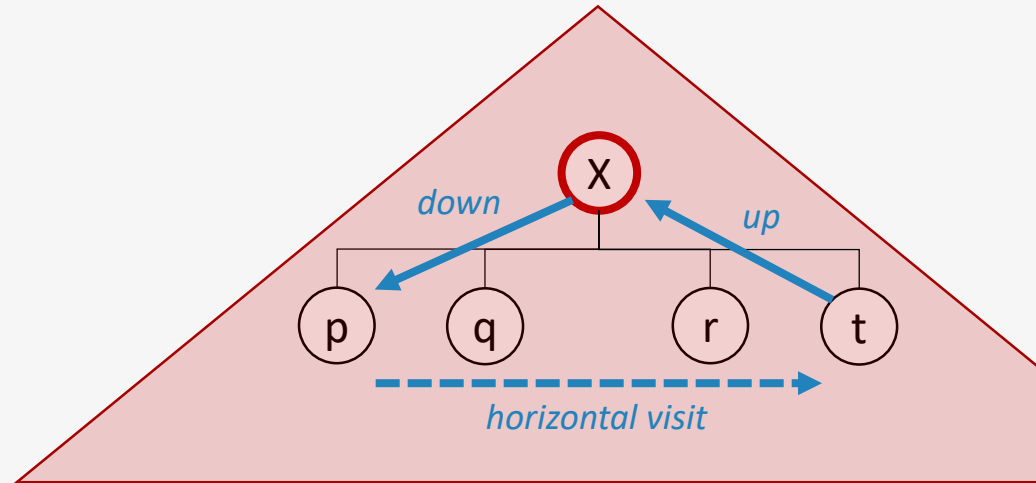
- Approach: define traversals as (normal) **strategy combinators**
- Idea: also parameterize “next” function to also support **right-to-left visits**



One-layer visits

layer s = down .* s .* up

layerOne s = layer (visitOne s)





Traversals

```
somewhere s = fix $ \x -> s .|. layerOne x
```

```
onceTd s     = fix $ \x -> s |> layerOne x     -- top down
```

```
onceBu s     = fix $ \x -> layerOne x |> s     -- bottom up
```

- Also: full traversals, spine traversals, innermost, outermost, etc.



Example trace

$\neg((p \vee q) \wedge \neg(p \wedge r))$

\Leftrightarrow De Morgan

$\neg(p \vee q)$ \vee $\neg\neg(p \wedge r)$

\Leftrightarrow De Morgan

$(\neg p \wedge \neg q) \vee$ $\neg\neg(p \wedge r)$

\Leftrightarrow Double Neg

$(\neg p \wedge \neg q) \vee (p \wedge r)$

Corresponding trace:

De Morgan at []

down

De Morgan at [0]

up

down

right

Double Neg at [1]

up

Summary for traversals:

- ✓ Simplify construction (traversals are first-class strategy combinators)
- ✓ Explicit representation (for better feedback)
- ✓ Many problem domains



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Conclusion



Trends and challenges

- **Authoring** intelligent tutoring system
 - Literature reports 200-300 authoring hours for 1 hour of instruction
 - We believe software technology can help
- **Data-driven** intelligent tutoring system
 - Use AI techniques to generate feedback from collected data
 - Raises questions about the role of expert domain knowledge
- Further **adaptation** and **personalization**
 - Models for mastery learning (e.g. Bayesian knowledge tracing)
- Designing tools for **less-structured problem domains**
 - For example, domains of software design and learning languages



Conclusion

- **Rewrite strategies** are used for feedback generation
- **Rewrite rules** can be embedded by using datatype-generic programming techniques
- **Generic traversals** can be composed from navigational actions and strategy combinators
- The presented approach can be applied to a wide range of **problem domains**

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Project website: <http://ideas.cs.uu.nl/>

Related publications

- Thomas van Noort, Alexey Rodriguez, Stefan Holdermans, Johan Jeuring, and Bastiaan Heeren. [A Lightweight Approach to Datatype-generic Rewriting](#). In *Proceedings of the ACM SIGPLAN Workshop on Generic Programming, WGP '08*, pages 13-24, 2008. ACM.
- Johan Jeuring, José Pedro Magalhães, and Bastiaan Heeren. [Generic Programming for Domain Reasoners](#). In Zoltán Horváth, Viktória Zsók, Peter Achten, and Pieter W. M. Koopman, editors, *Proceedings of the Tenth Symposium on Trends in Functional Programming, TFP 2009*, pages 113-128, 2009. Intellect.
- Thomas van Noort, Alexey Rodriguez yakushev, Stefan Holdermans, Johan Jeuring, Bastiaan Heeren, and José Pedro Magalhães. [A Lightweight Approach to Datatype-generic Rewriting](#). *J. Funct. Program.*, 20(3-4):375-413, 2010.
- Bastiaan Heeren and Johan Jeuring. [Feedback services for stepwise exercises](#). *Science of Computer Programming*, 88:110-129, 2014. Software Development Concerns in the e-Learning Domain.
- Josje Lodder, Bastiaan Heeren and Johan Jeuring. [A Domain Reasoner for Propositional Logic](#). *Journal of Universal Computer Science*, 22(8):1097-1122, 2016.
- Bastiaan Heeren and Johan Jeuring. [An Extensible Domain-Specific Language for Describing Problem-Solving Procedures](#). In Elisabeth André, Ryan Baker, Xiangen Hu, Ma. Mercedes T. Rodrigo, and Benedict du Boulay, editors, *Proceedings of Artificial Intelligence in Education: 18th International Conference, AIED 2017*, pages 77-89, 2017. Springer International Publishing.
- Josje Lodder, Bastiaan Heeren, Johan Jeuring, and Wendy Neijenhuis. [Generation and Use of Hints and Feedback in a Hilbert-Style Axiomatic Proof Tutor](#). *International Journal of Artificial Intelligence in Education*, 2020.